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A CONSERVATIVE SPLIT-EXPLICIT INTEGRATION SCHEME WITH FOURTH ORDER HORIZONTAL ADVECTION

by
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A conservative split-explicit integration scheme with fourth order horizontal advection

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SUMMARY

A split-explicit finite difference scheme is developed which combines the accuracy and economy required for numerical weather prediction with the conservation properties required for climate change experiments. Results are presented showing that the aims of the scheme are achieved.

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1. INTRODUCTION

While spectral methods are in almost universal use for the global atmospheric models used for weather forecasting and climate research, there is still an interest in developing finite difference methods suitable for these applications. This is because, as resolution increases, the cost of the spectral method may increase faster than that of finite difference methods. Also, as interest focusses on local behaviour in the results, finite difference methods may be found better at representing local, rather discontinuous behaviour because they avoid problems associated with spectral fitting.

A very efficient finite difference scheme developed for use in forecast models is the split-explicit method, Gadd (1978a). Gadd combined this method with an accurate version of the Lax-Wendroff advection scheme, Gadd (1978b). Mesinger (1981) used the split-explicit method together with the Heun advection scheme, which avoids the complexity of time-staggering. Both schemes have been used successfully in operational models for a number of years; the former has been used in the U.K. Meteorological Office global operational model since 1982.

In climate research an important requirement is to conserve mass-weighted temperature and moisture, so that the heat and moisture budgets can be accurately calculated and their effect in various climate change scenarios assessed. Neither of the above split-explicit schemes do this when written in the standard sigma or hybrid sigma/pressure vertical coordinate systems. The difficulty is that the vertical advection of potential temperature forms part of the gravity wave dynamics and has to be treated separately from the horizontal advection. Correct treatment of the angular momentum and energy budgets is also important. Simmons and Burridge (1981) show how to do this in hybrid coordinates. For energy

conservation, it is necessary for the calculation of vertical velocity to be consistent with that of the horizontal velocity through the continuity equation. This is also not ensured by the standard split-explicit method.

It is not generally agreed what properties of the finite difference schemes are necessary to ensure satisfactory results in extended integrations. Lagrangian conservation properties may be important, but cannot be enforced within conventional methods. Conservation of quadratic quantities provides an approximation to Lagrangian conservation. Mesinger (1981) has shown how this can be achieved in split-explicit finite difference schemes.

In this paper, the split-explicit method is modified to conserve mass-weighted potential temperature and moisture, and to treat the energy conversion term consistently.

This is done by treating only the vertical advection of a basic state temperature with a short timestep, and treating the remainder of the advection with a conservative scheme and a long timestep, using the average velocity from the short timesteps. The technique is exactly parallel to the split used in semi-implicit models, Simmons et al. (1978). It is also necessary to ensure that a finite difference flux conservation law can be obtained by combining the advection scheme with the continuity equation in finite difference form. This is done by using the Heun scheme, since this combination is difficult to achieve with the time-staggered Lax-Wendroff scheme. The scheme is implemented in hybrid coordinates as in Simmons and Burridge (1981) to ensure correct treatment of the angular momentum and enery.

2. THE FORECAST EQUATIONS

To simplify the presentation, the scheme is described in horizontal Cartesian coordinates (x,y). When used in a global model the application on a latitude longitude grid in spherical polar coordinates is straightforward. Careful attention has to be paid to the method of filtering near the poles to ensure that the conservation properties are retained, the method required is discussed at the end of section 3. The construction of conservation laws depends on the continuity equation, whose form depends on the choice of vertical coordinate. We use the hybrid coordinate system described by Simmons and Burridge (1981), which combines the advantages of the terrain-following sigma coordinate system near the surface, and the pressure system in the stratosphere, thereby reducing the error in the pressure gradient term. Only the adiabatic equations are presented, since moisture conservation is assured by using the same advection scheme as for the perturbation potential temperature.

Define a vertical coordinate $\eta = \eta(p,p_*)$, where $\eta(0,p_*)=0$ and $\eta(p_*,p_*)=1$. The equations are then

$$\partial u/\partial t + u\partial u/\partial x + v\partial u/\partial y + \dot{\eta}\partial u/\partial \eta + \partial \phi/\partial x + (RT/p)\partial p/\partial x - fv = F_u \qquad (1)$$

$$\partial v/\partial t + u\partial v/\partial x + v\partial v/\partial y + \dot{\eta}\partial v/\partial \eta + \partial \phi/\partial y + (RT/p)\partial p/\partial y + fu = F_v \qquad (2)$$

$$\partial \theta/\partial t + u\partial \theta/\partial x + v\partial \theta/\partial y + \dot{\eta}\partial \theta/\partial \eta = F_\theta \qquad (3)$$

$$\partial/\partial \eta(\partial p/\partial t) + \partial/\partial x(u\partial p/\partial \eta) + \partial/\partial y(v\partial p/\partial \eta) + \partial/\partial \eta(\dot{\eta}\partial p/\partial \eta) = 0 \qquad (4)$$

The quantities F_u, F_v, F_θ represent source terms, and also include any diffusion required for computational reasons. The vertical boundary conditions are:

$$\dot{\eta} = 0 \text{ at } \eta = 0,1. \tag{5}$$

Integrating (4) in the vertical from η=0 to 1 gives:

$$\partial p_*/\partial t = -\int_0^1 [\partial/\partial x(u\partial p/\partial \eta) + \partial/\partial y(v\partial p/\partial \eta)]d\eta \tag{6}$$

Integrating (4) from η=0 to η gives:

$$\dot{\eta}\partial p/\partial \eta = -\partial p/\partial t - \int_0^\eta [\partial/\partial x(u\partial p/\partial \eta) + \partial/\partial y(v\partial p/\partial \eta)]d\eta \tag{7}$$

The hydrostatic relation is given by:

$$\partial \Phi/\partial \eta = -(RT/p)\partial p/\partial \eta = -c_p\theta\partial \Pi/\partial \eta \quad , \tag{8}$$

where $\Pi=(p/1000)^\kappa$

3. THE INTEGRATION SCHEME

The variables are held on the Arakawa 'B' grid. The variables u, v, θ and Φ are held at levels η_k, where k is the vertical grid-length index, while η is held at the intermediate levels $\eta_{k+1/2}$. The lower boundary is $k=^1/_2$ and the upper boundary $k=TOP+^1/_2$. The pressure is defined at intermediate levels by

$$p_{k+1/2} = A_{k+1/2} + B_{k+1/2}p_*, \tag{9}$$

where $A_{k+1/2}$ and $B_{k+1/2}$ are specified constants. Thus

$$(\partial p/\partial p_*)_{k+1/2} = B_{k+1/2} \tag{10}$$

and

$$\Delta p_k = (A_{k+1/2}-A_{k-1/2}) + (B_{k+1/2}-B_{k-1/2})p_* \tag{11}$$

Note that this definition makes Δp_k negative, since k increases with physical height. In the split explicit integration scheme, the solution procedure is split into two parts, called the 'adjustment' and 'advection' steps. The adjustment timestep is written as δt, the advection timestep as Δt. In the former, the pressure, temperature, and wind fields are updated using the pressure gradient and the Coriolis terms, and the vertical advection of the basic state potential temperature. Only the final updated values of surface pressure and horizontal wind are used in the next step. The average horizontal wind from the adjustment step is used to define the horizontal advection in the advection step, and, via the continuity equation, the vertical advection. This procedure is needed to ensure conservation and correct treatment of the energy conversions. All advection increments except the vertical advection of basic state potential temperature are then calculated in the advection step, together with any horizontal diffusion.

The standard finite difference notation

$$\delta_x X = (X(x+{}^1/_2\Delta x) - X(x-{}^1/_2\Delta x))/\Delta x$$

$$\overline{X}^x = {}^1/_2(X(x+{}^1/_2\Delta x) + X(x-{}^1/_2\Delta x))$$

is used.

(a) The adjustment step

This uses the 'forward-backward' scheme in which a forward step is used for the u and v equations, and the new values of these variables are then used in the p_* and θ equations. The 'forward' part of the integration scheme is:

$$u_k^{n+1} = u_k^n + \delta t\Big[1/2f(v_k^n + v_k^{n+1}) - \overline{\{\delta_x \Phi_k^n + \frac{c_p \overline{\theta_k}^x}{(\kappa+1)} \delta_x\Big[\frac{\Pi_{k+1/2}p_{k+1/2} - \Pi_{k-1/2}p_{k-1/2}}{\Delta p_k}\Big]^n\}}^y\Big], \quad (12)$$

$$v_k^{n+1} = v_k^n - \delta t\Big[1/2f(u_k^n + u_k^{n+1}) + \overline{\{\delta_y \Phi_x^n + \frac{c_p \overline{\theta_k}^y}{(\kappa+1)} \delta_y\Big[\frac{\Pi_{k+1/2}p_{k+1/2} - \Pi_{k-1/2}p_{k-1/2}}{\Delta p_k}\Big]^n\}}^x\Big], \quad (13)$$

Equations (12) and (13) can be arranged to allow explicit integration. The hydrostatic equation is approximated by

$$\Phi_k = \Phi_* + \sum_{m-1}^{k-1} c_p\theta_m(\Pi_{m+1/2} - \Pi_{m-1/2})$$
$$c_p\theta_k\Big(\Pi_{k-1/2} - \frac{(\Pi_{k-1/2}p_{k-1/2} - \Pi_{k+1/2}p_{k+1/2})}{(\kappa+1)\Delta p_k}\Big), \quad (14)$$

The special form of the last term is chosen to ensure angular momentum conservation. The implicit treatment of the Coriolis term removes the stability problem noted by Gadd (1980).

The 'backward' part of the integration scheme is given by

$$p_*^{n+1} = p_*^n - \delta t \sum_{m=1}^{TOP} D_m^{n+1} \quad (15)$$

$$\theta^{n+1} = \theta^n - \frac{\delta t}{2(\Delta p)^n_k}\Big[\Big(\dot{\eta}\frac{\partial p}{\partial \eta}\Big)^{n+1}_{k+1/2}(\theta_{Rk+1} - \theta_{Rk}) + \Big(\dot{\eta}\frac{\partial p}{\partial \eta}\Big)_{k-1/2}(\theta_{Rk} - \theta_{Rk-1})\Big] \quad (16)$$

where $\theta_R(\eta)$ is a basic state profile of θ. As discussed by Simmons et al. (1978), this must be carefully chosen to ensure computational stability. The problem is caused by instability of gravity waves, rather than the energetic inconsistency of the adjustment equations considered on their own. The standard choice, used here, is an isothermal basic state with temperature 300°K and surface pressure 1000hpa. The form of these equations ensures that mass and mass-weighted potential temperature are conserved; in particular the integral of $\Delta p^n \theta^{n+1}$ equals that of $\Delta p^n \theta^n$.

$$\left(\frac{\dot{\eta}\partial p}{\partial \eta}\right)_{k+1/2} = \left(\frac{\partial p}{\partial p_*}\right)_{k+1/2} \sum_{M=1}^{TOP} D_m - \sum_{m=1}^{k} D_m \tag{17}$$

$$D_m = \overline{[\delta_x(u_m \overline{\Delta p_m}^{xy})}^{y} + \overline{\delta_y(v_m \overline{\Delta p_m}^{xy})}^{x}], \tag{18}$$

In order to ensure that θ is conserved under advection, it is necessary that all advection is done by a three-dimensional velocity field which satisfies the continuity equation. The average values of $u_m \Delta p_m$ and $v_m \Delta p_m$ over the adjustment steps must be saved for use in the advection step.

(b) The advection step

The basic state θ_R is first subtracted from θ to give θ'. The Heun advection scheme is used, as in Mesinger (1981). This avoids the stability problem noted by Gadd (1980). Experiments within the split-explicit model described by Bell and Dickinson (1987) have shown that it is more stable than the Lax-Wendroff scheme used in that model, even though it has growing eigensolutions of order $(1+O(\Delta t^4))$. The scheme can be corrected to remove this instability. The correction term can be found by a Taylor expansion, and is scaled by $(U\Delta t/\Delta x)^3$. It is thus only effective when $U\Delta t/\Delta x$ is close

to 1, and is found in practice to be submerged by the diffusion required to handle the cascade of energy to small scales. It is therefore not used here.

The scheme has two steps. The advecting velocity for both is the average value saved from the adjustment steps.

Define

$$\underline{U}_k = (U_k, V_k) = (u_k \overline{\overline{\Delta p_k}}^{xy}, \; v_k \overline{\overline{\Delta p_k}}^{xy}), \tag{19}$$

as saved from the adjustment steps. Define

$$\left(\dot{\eta}\frac{\partial p}{\partial \eta}\right)_{k+1/2} \equiv E_{k+1/2} \tag{20}$$

where $E_{k+1/2}$ is calculated from the finite difference formulae (17) and (18). Use the $\hat{}$ overbar to denote values at the end of the first advection step. The finite difference equations are then:

$$\begin{aligned}
\Delta p_k^n \hat{\theta}'_k \; &= \; \Delta p_k^n \theta_k^n - \\
&\Delta t\{(1+\nu)\overline{\overline{U_m}^{y}}^{x}\delta_x\theta'_k - \nu\overline{\overline{U_m}^{y}}^{3x}\delta_x\theta'_k + (1+\nu)\overline{\overline{V_m}^{x}}^{y}\delta_y\theta'_k - \nu\overline{\overline{V_m}^{x}}^{3y}\delta_y\theta'_k\} \\
&- {}^1/_2\Delta t\{E_{k+1/2}(\theta'_{k+1}-\theta'_k) + E_{k-1/2}(\theta'_k-\theta'_{k-1})\}
\end{aligned} \tag{21}$$

$$\begin{aligned}
(\Delta p)_k^n \hat{u}_k \; &= \; (\Delta p)_k^n u_k^n - \\
&\Delta t\{(1+\nu)\overline{\overline{U_m}^{xy}}^{x}\delta_x u_k - \nu\overline{\overline{U_m}^{xy}}^{3x}\delta_x u_k + (1+\nu)\overline{\overline{V_m}^{xy}}^{y}\delta_y u_k - \nu\overline{\overline{V_m}^{xy}}^{3y}\delta_y u_k\}. \\
&- {}^1/_2\Delta t\{\overline{E_{k+1/2}}^{xy}(u_{k+1}-u_k) + \overline{E_{k-1/2}}^{xy}(u_k-u_{k-1})\},
\end{aligned} \tag{22}$$

with a similar equation for v.

Note that in the scheme of Gadd (1978b), higher accuracy is achieved without requiring the timestep to be reduced by modifying the second step of the Lax-Wendroff scheme. In the Heun scheme, it is necessary to use the same finite difference approximation in both steps, or else there is an $O(\Delta t^2)$ instability. The value $\nu=1/6$ in equations (21) to (22) gives fourth order accuracy, but will increase the squared amplification rate of the growing solution from $(1+{}^1/_4\xi^4)$ to $(1+{}^1/_4\xi^4_1)$ where ξ is the Courant number and $\xi_1=1.37\xi$. This will reduce the maximum timestep that can safely be used. A fixed value must be used for ν to allow conservation, but for applications where this is not important the choice $\nu=1/6(1-\xi^2)$ should avoid the need to reduce the timestep.

The second advection step can be written:

$$(\Delta p)_k^{n+1}\ \theta_k'^{n+1} = \{{}^1/_2(\Delta p)_k^n(\theta'^n - \hat{\theta}')_k + {}^1/_2(\Delta p)_k^{n+1}(\theta'^n + \hat{\theta}'_k)_k\} -$$

$${}^1/_2\Delta t(\underline{U}.\nabla\theta_k'^n + \underline{U}.\nabla\hat{\theta}'_k). \qquad (23)$$

where $\underline{U}$ is the three-dimensional velocity vector. The equation for u is

$$\overline{\overline{(\Delta p)_k^{n+1}u_k^{n+1}}}^{xy} = \{{}^1/_2\overline{(\Delta p)_k^n(u^n-\hat{u})_k}^{xy} + {}^1/_2\overline{(\Delta p)_k^{n+1}(u^n+\hat{u})_k}^{xy}\} - \qquad (24)$$

$${}^1/_2\Delta t\ (\underline{U}.\nabla u_k^n + \underline{U}.\nabla\hat{u}_k)$$

with a similar equation for v. The form of equations (23) and (24) ensures conservation under time differencing.

(c) Fourier filtering

When this finite difference scheme is used in a global model on a latitude-longitude grid, some form of filtering is needed at high latitudes to avoid the need for a very short timestep. It is necessary to ensure that global conservation properties are not affected by the filtering. Mass-weighted increments to θ, and mass weighted velocity fields $\Delta p(u,v)$

are therefore filtered. Filtering mass-weight velocity fields before the update to p_* removes the need to filter p_* and θ increments after the adjustment steps, so that the conservation proofs of section 4 do not have to consider the effect of filtering. This strategy also avoids the problem of filtering fields which vary rapidly along a model coordinate surface.

(d) Conservation properties

The angular momentum and energy conservation properties are retained with respect to vertical differencing by similar arguments to those of Simmons and Burridge (1981), modified by the use of potential temperature rather than temperature as the model variable.

We set out the proof that the global mass-weighted mean of θ is conserved under meridional advection as an example of how it works for other variables and directions. Combining (17-20) gives the continuity equation in the form:

$$E_{k+1/2} = \left(\frac{\partial p}{\partial p_*}\right)_{k+1/2} \sum_{m-1}^{TOP} D_m - \sum_{m-1}^{k} D_m \qquad (25)$$

where

$$D_m = \Delta_y \overline{V_m}^x \ , \qquad (26)$$

A simple second order forward update of θ by meridional advection, and advection by the vertical motion associated with the meridional motion, is given by

$$\Delta p_k^+ \theta_k^+ = \Delta p_k^+ \theta_k - \Delta t[\overline{V_k \delta_y \theta_k}^{xy} +$$

$$^1/_2\{E_{k+1/2}(\theta_{k+1}-\theta_k) + E_{k-1/2}(\theta_k-\theta_{k-1})\}] \ , \qquad (27)$$

where the superscript + denotes updated values.

The update of p_* can be written

$$p_*^+ = p_* - \Delta t \sum_{m-1}^{TOP} D_m, \tag{28}$$

because of the definition of V_m as the average over the adjustment steps. Equation (10) can be used to rewrite (28) as

$$E_{k+1/2} - E_{k-1/2} = \Delta B_k (p_*^+ - p_*)/\Delta t - D_k. \tag{29}$$

Multiplying (29) by θ_k, substituting for Δp_k using (11), and adding to (27) gives

$$(\Delta A_k + \Delta B_k p_*^+)(\theta_k^+ - \theta_k) + \theta_k (p_*^+ - p_*)\Delta B_k =$$

$$-\Delta t[\underline{V_k \delta_y \theta_k}^{y}_{x} + \underline{\theta_k \delta_y V_k}^{x} +$$

$$^1/_2\{E_{k+1/2}(\theta_{k+1} - \theta_k) + (E_{k-1/2}(\theta_k - \theta_{k-1}) + 2\theta_k(E_{k+1/2} - E_{k-1/2})\}] \tag{30}$$

This reduces to

$$(\Delta p_k \theta_k)^+ - \Delta p_k \theta_k = -\Delta t[\underline{\delta_y V_k \theta_k}^{x\,y} +$$

$$^1/_2\{E_{k+1/2}(\theta_{k+1} + \theta_k) - E_{k-1/2}(\theta_k + \theta_{k-1})\}], \tag{31}$$

which gives the desired conservation integral when integrated over y.

Now consider the fourth order terms in (21) and (22). Conservation cannot be achieved if ν is a function of ξ, as may be necessary to avoid reducing the timestep. Suppose that ν is a constant. The terms

$$(1+\nu)\underline{U_k \delta_x \theta_k}^{x}_{y} - \nu \underline{U_k \delta_x \theta_k}^{3x}_{y}$$

can be expanded as

$$\overline{(1+\nu)U_k}^{y}(x+{}^1/_2\Delta x)(\theta_k(x+\Delta x)-\theta_k(x)) -$$

$$\overline{\nu U_k}^{y}(x+3/2\Delta x)(\theta_k(x+2\Delta x)-\theta_k(x+\Delta x)), \qquad (32)$$

with symmetrical terms in $-\Delta x$. These terms cancel with contributions from $\theta_k^+(x+\Delta x)$ and $\theta_k^+(x-\Delta x)$ when $p_*^+\theta_k^+$ is summed over x to give the required conservation.

We also demonstrate that the integral of $\Delta p\theta^2$ is conserved using the second order accurate approximation to the advection terms. Multiply (29) by θ_k^2 and add to (27) multiplied by $2\theta_k$:

$$2\Delta p_k^+\theta_k[\theta_k^+ - \theta_k] + \theta_k{}^2(p_* - p_*)\Delta B_k =$$

$$- \Delta t\ [\{2\theta_k\overline{V_k}^{x}\delta_y\theta_k + \theta_k\delta_y\overline{V_k}^{x}\} + \{\theta_k\overline{E_{k+1/2}}^{y}(\theta_{k+1} - \theta_k) +$$

$$\theta_k\overline{E_{k-1/2}}^{y}(\theta_k - \theta_{k-1}) + \theta_k^2(E_{k+1/2} - E_{k-1/2})\}]. \qquad (33)$$

The left hand side is a discrete approximation to

$$\Delta p_k\frac{\partial(\theta_k^2)}{\partial t} + \theta_k^2\ \frac{\partial\Delta p_k}{\partial t}. \qquad (34)$$

However, it cannot be written as exact conservation of $\Delta p_k\theta_k^2$. The right hand side becomes

$$-\Delta t[\ \overline{\delta_y\overline{V_k}^{y}}^{x}(2(\theta_k)^2 - \theta_k{}^2) + \theta_k\theta_{k+1}\overline{E_{k+1/2}}^{y} - \theta_k\theta_{k-1}\overline{E_{k-1/2}}^{y}]. \qquad (35)$$

This is in conservation form. In order to achieve quadratic conservation with the fourth order terms included, the E_k's must be redefined (M. Fisher, private communication). The resulting scheme is rather less accurate because it uses a broader stencil of gridpoints.

4. RESULTS

The proposed scheme has been implemented for both forecast and climate applications. Some illustrative results are given, but exact comparisons with previous schemes used in the Meteorological office are not possible since many other changes were made in migrating the models to a new computer system.

The proposed forecast configuration is a 288 x 217 latitude-longitude grid with 20 levels, the highest at 7 hpa. The performance is illustrated by a 6 day forecast from the 10th of September 1990 (Fig. 1). The verifiying analysis is shown in Fig. 2 and a forecast from the operational model described by Bell and Dickinson (1987) in Fig. 3. The results show that the modified scheme still works satisfactorily in forecast mode.

The proposed climate model configuration is a 96 x 73 latitude-longitude grid. The levels are the same as in the forecast configuration. The performance is illustrated by monitoring the kinetic energy and the variance in the temperature field. Graphs against time of total, zonal, and eddy kinetic energy, and of temperature variance, all integrated globally, are shown in Fig. 4. Sample values of the first three from the current operational climate model are 151 m^2s^{-2}, 95 m^2s^{-2} and 56 m^2s^{-2}. The results show that a model including the proposed finite difference scheme gives comparable figures. Checks on the conservation properties, using a CRAY Y-MP computer, showed that the temperature error was about 1°K in 15 years, with comparable results for other variables.

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LIST OF FIGURES

Fig. 1 6 day PMSL forecast (hpa) verifying at 0Z on 16 September 1990, using the proposed integration scheme.
Contour interval 4 hpa.

Fig. 2 Verifying analysis for 0Z on 16 September 1990.

Fig. 3 6 day PMSL forecast verifying at 0Z on 16 September 1990 using current Meteorological Office operational model.

Fig. 4 Evolution over a 30 day forecast of global means of

(A) Total kinetic energy (m^2s^{-2})

(B) Zonal kinetic energy (m^2s^{-2})

(C) Eddy kinetic energy (m^2s^{-2})

(D) Variance of temperature (K^2)

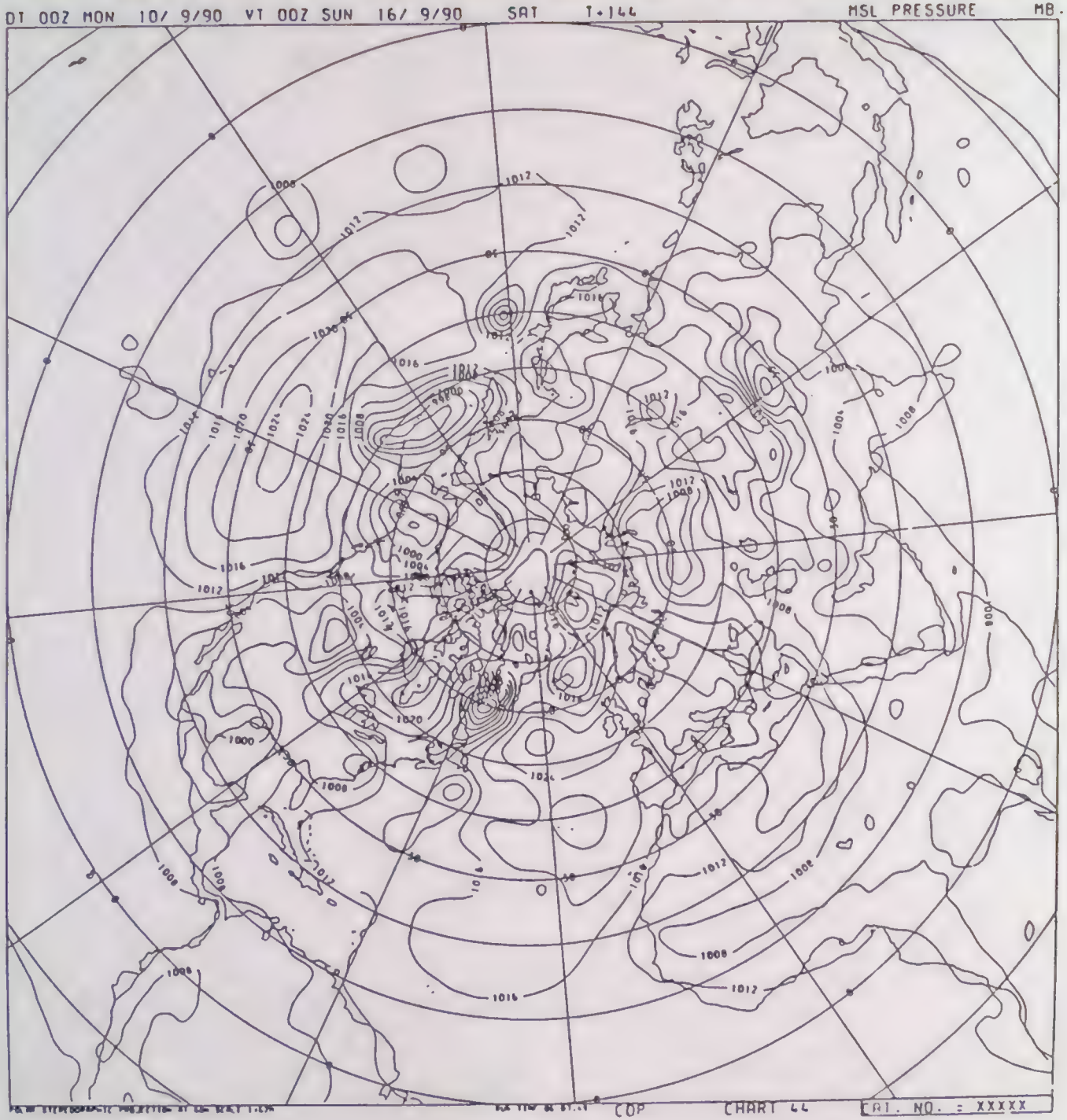
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Fig. 1

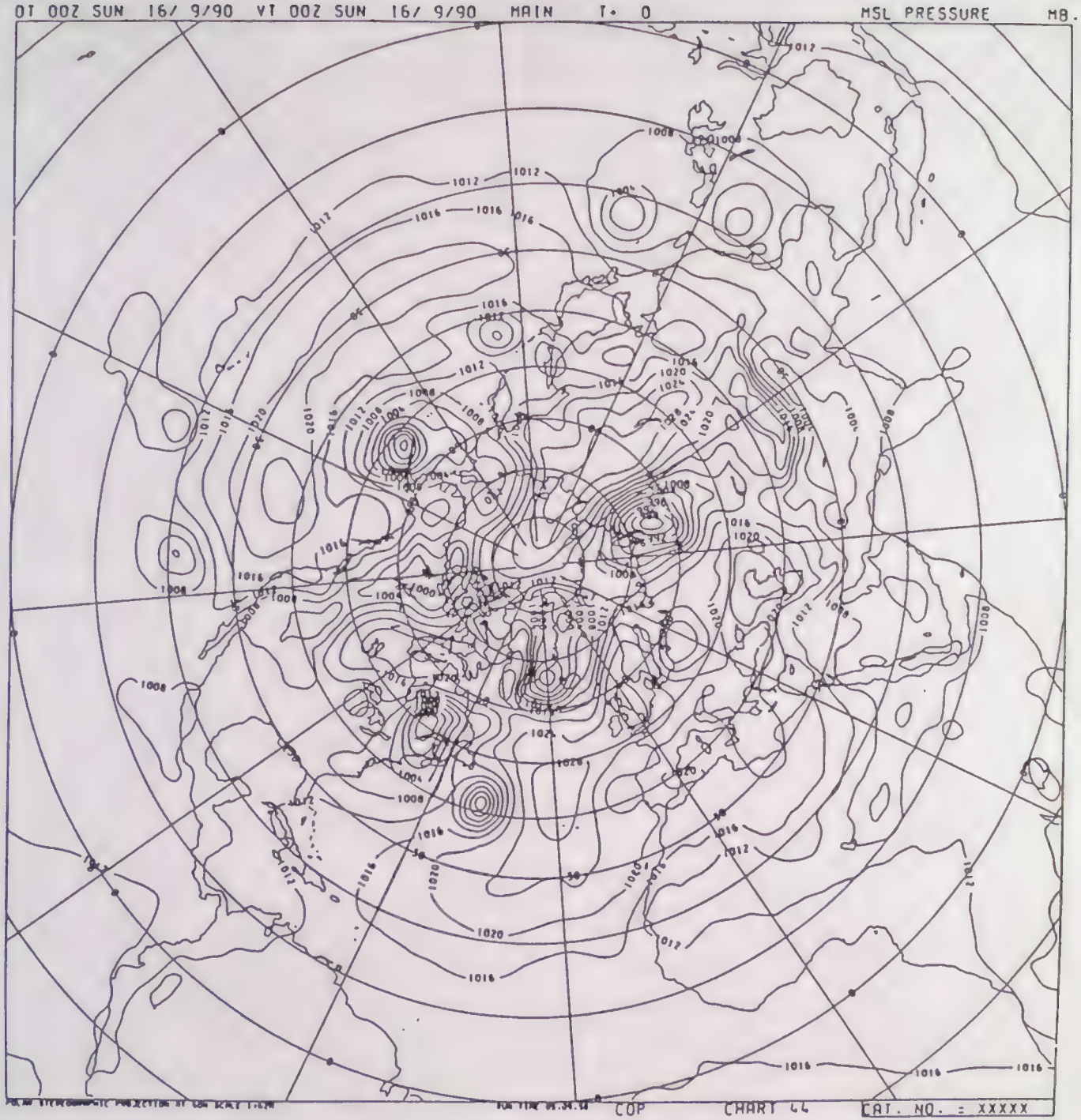
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Fig. 2

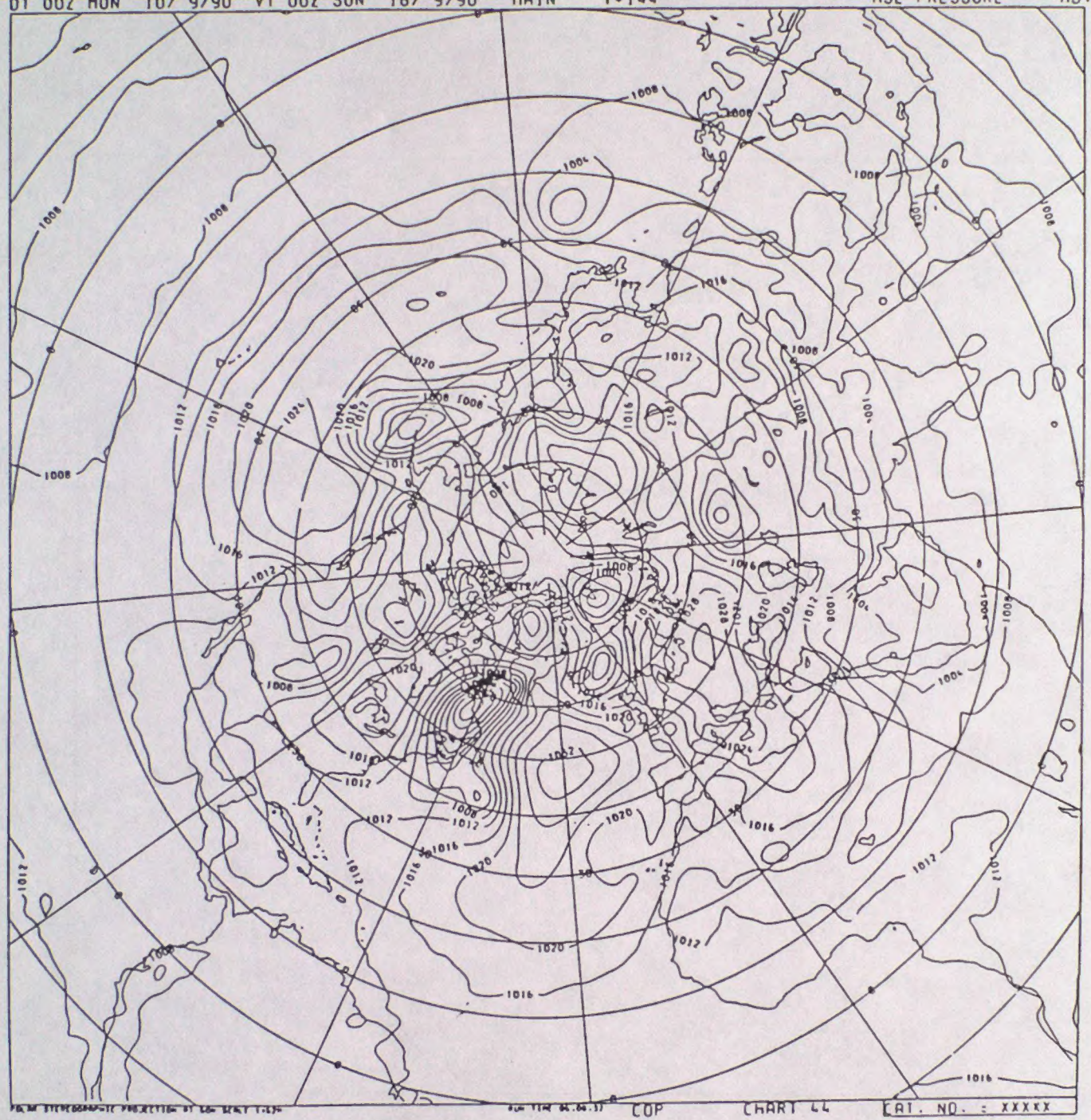
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Fig. 3

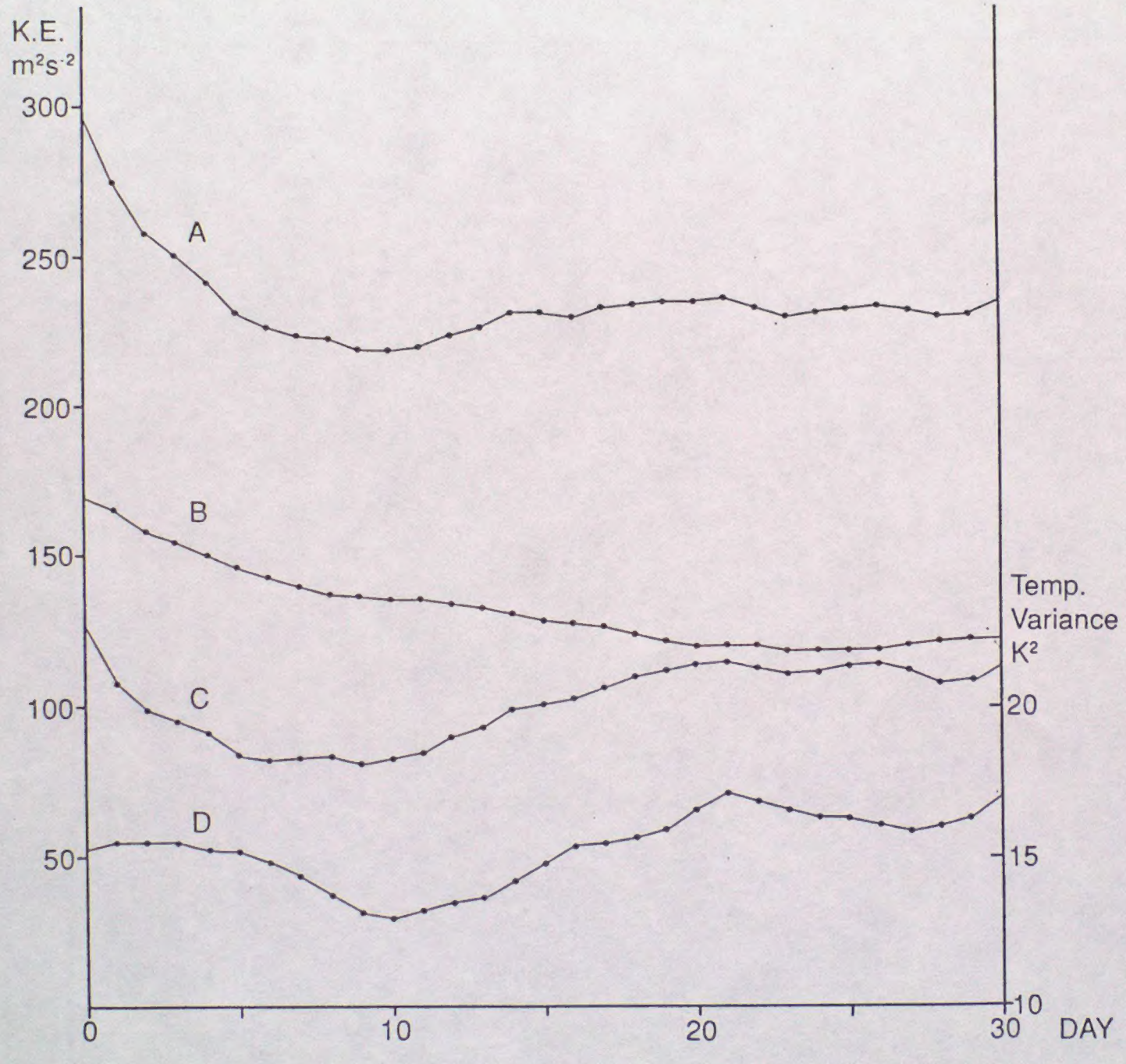
K.E.
m²s⁻²
300
250
200
150
100
50
A
B
C
D
Temp.
Variance
K²
20
15
10
0
10
20
30
DAY

Fig. 4

CURRENT SHORT - RANGE FORECASTING RESEARCH SCIENTIFIC NOTES (NOVEMBER 1990)

The Short- Range Forecasting Research Branch (formerly Met O 11) Scientific Notes which contain information of current use are listed below. The complete set of Scientific Notes is available from the National Meteorological Library on loan, if required.

1. **The theory of periodic solutions of the semi-geostrophic equations.**
R.J. Purser
October 1987

2. **Properties of the partial differential equations governing various types of atmospheric motions and implications for numerical methods.**
M.J.P. Cullen
December 1987

3. **A geometric model of balanced, axisymmetric flows with embedded penetrative convection.**
G.J. Shutts, M. Booth and J. Norbury
February 1988

4. **Implicit finite difference methods for computing discontinuous atmospheric flows.**
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5. **Variational aspects of semi-geostrophic theory.**
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6. **On the incorporation of atmospheric boundary layer effects into a balanced model.**
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7 **Implicit finite difference methods for modelling discontinuous atmospheric flows.**
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16. A conservative split-explicit integration scheme suitable for forecast and climate models.
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17. The dynamical structure of some North Atlantic depressions simulated by the fine-mesh model.
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18. An Idealised Simulation of the Indian Monsoon using Primitive Equation and Quasi - Equilibrium models.
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19. Time - Integration Schemes for meteorological simulations involving oscillatory and dissipative modes.
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20. Mesoscale Model Experimental forecasts of the Haar of North East Scotland.
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21. A conservative split-explicit integration scheme with fourth order horizontal advection.
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